Donald E. Kunth

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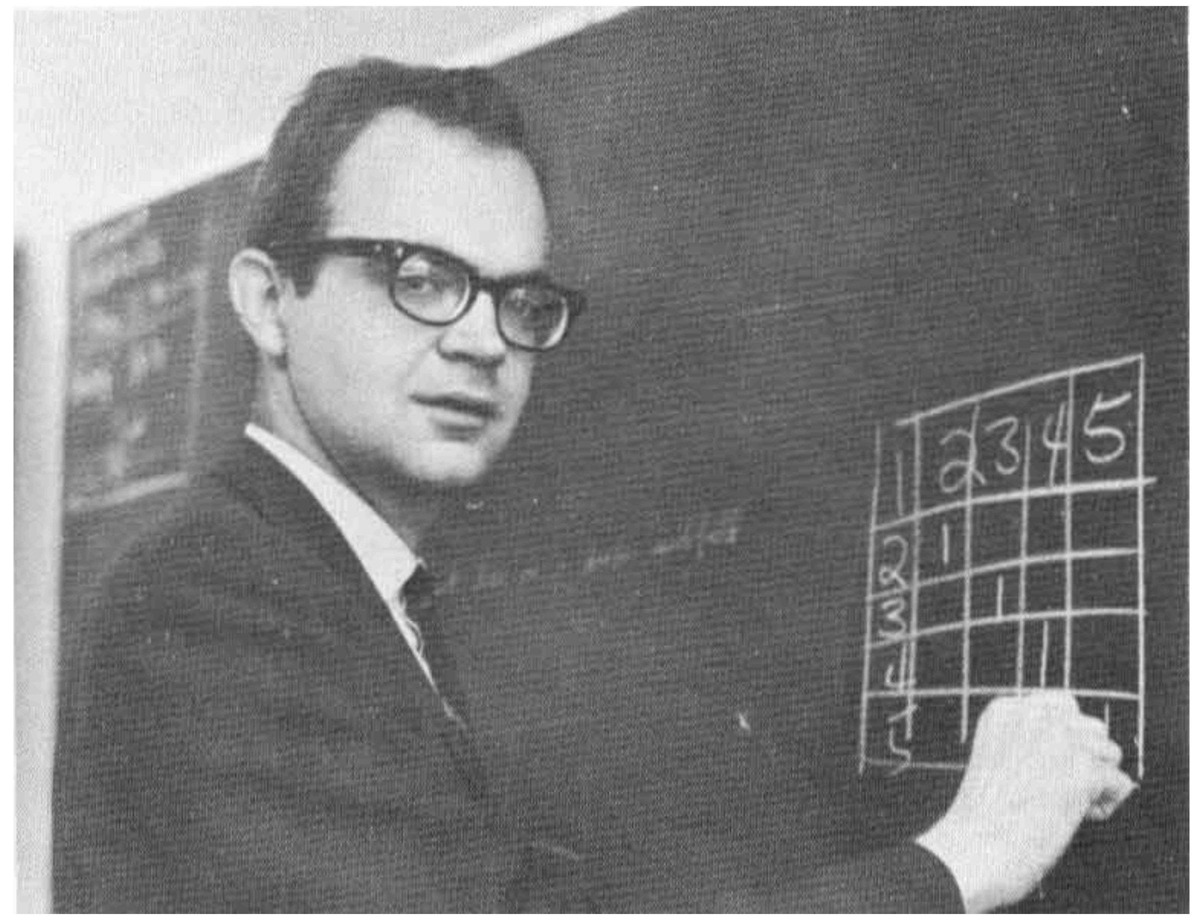
Donald Ervin Knuth (Ka-NOOTH)

Essentials:
- TeX
- up-arrow notation (↑)
- Knuth–Morris–Pratt algorithm (KMP string search)
- 125,518 Citations on Google Scholar
- Recipient of the ACM Turing Award
- CV 42 pages
- Called “father of the analysis of algorithms”
- Still alive (b. 1938, Wisconsin, U.S., c. age 84), still writing.

https://www-cs-faculty.stanford.edu/~knuth/

Education and Personal Life



- He earned simultaneously B.S. and M.S. in mathematics from Case Institute of Technology 1960 by a special award of the faculty, who considered his work exceptionally outstanding
- Ph.D. in mathematics from California Institute of Technology (Caltech) 1963; title: Finite Semifields and Projective Planes
- During his years as a graduate student Knuth married Nancy Jill Carter on 24 June 1961. Their two children John M. Knuth and Jennifer S. Knuth were born in 1965 and 1966 respectively.
- Professor Emeritus of The Art of Computer Programming at Stanford University

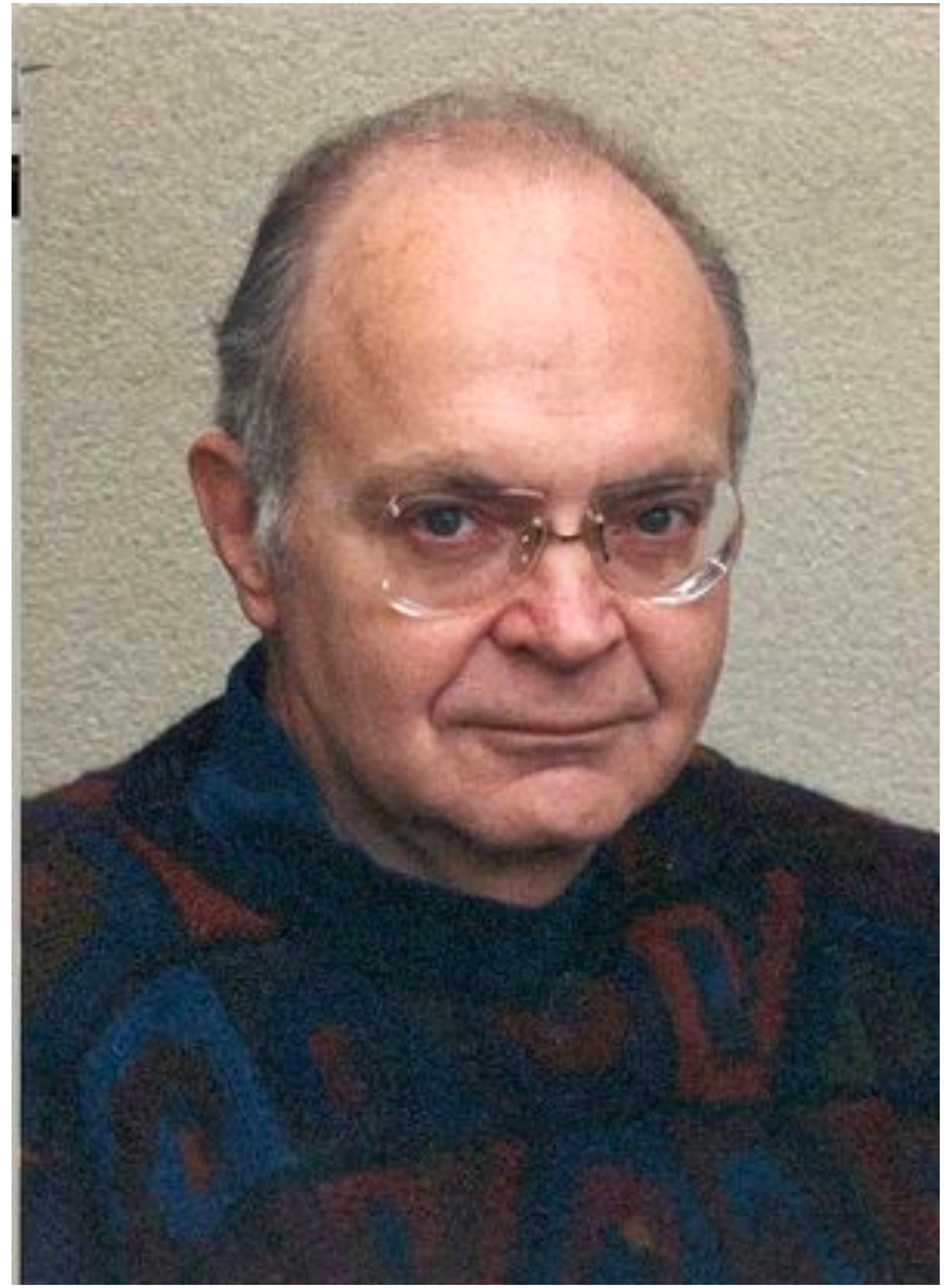
His work

His main life's work has been to write The Art of Computer Programming, a work-still-in-progress that attempts to organize and summarize what is known about the vast subject of computer methods and to give it firm mathematical and historical foundations. (The four volumes published so far have been translated into many languages and more than a million copies have been sold.)

As a researcher in computer science, he is the “father" of several subareas called the analysis of algorithms, attribute grammars, empirical study of programming languages, and literate programming.

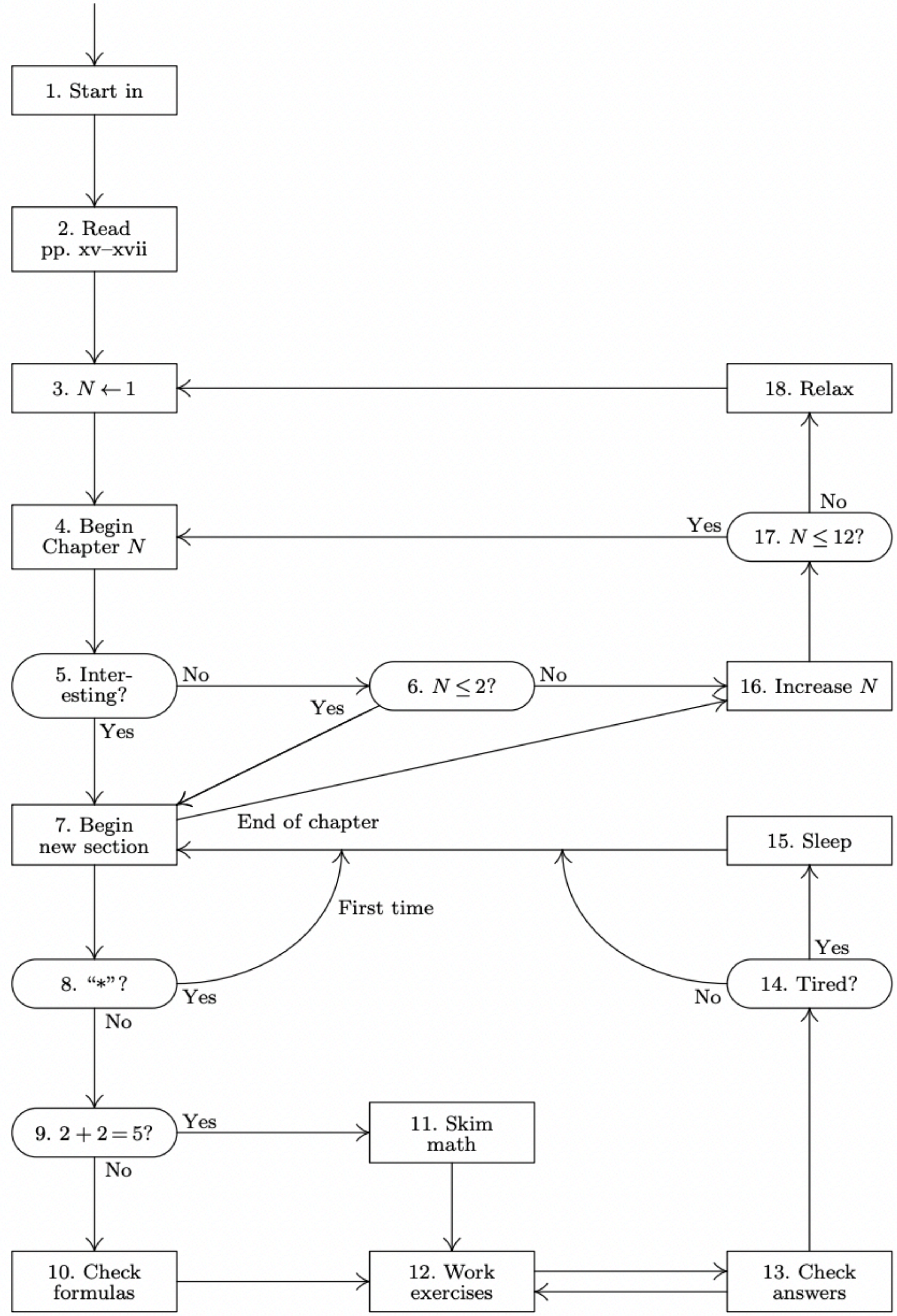
His best-known research in mathematics is represented by the Knuth--Bendix algorithm for word problems, the Robinson--Schensted--Knuth correspondence between matrices and tableaux, and an analysis of the big bang that occurs in the evolution of random graphs.

And as a programmer, he wrote software systems called TeX and MF that are used for the majority of today's mathematical publications.



The Art of Computer Programming

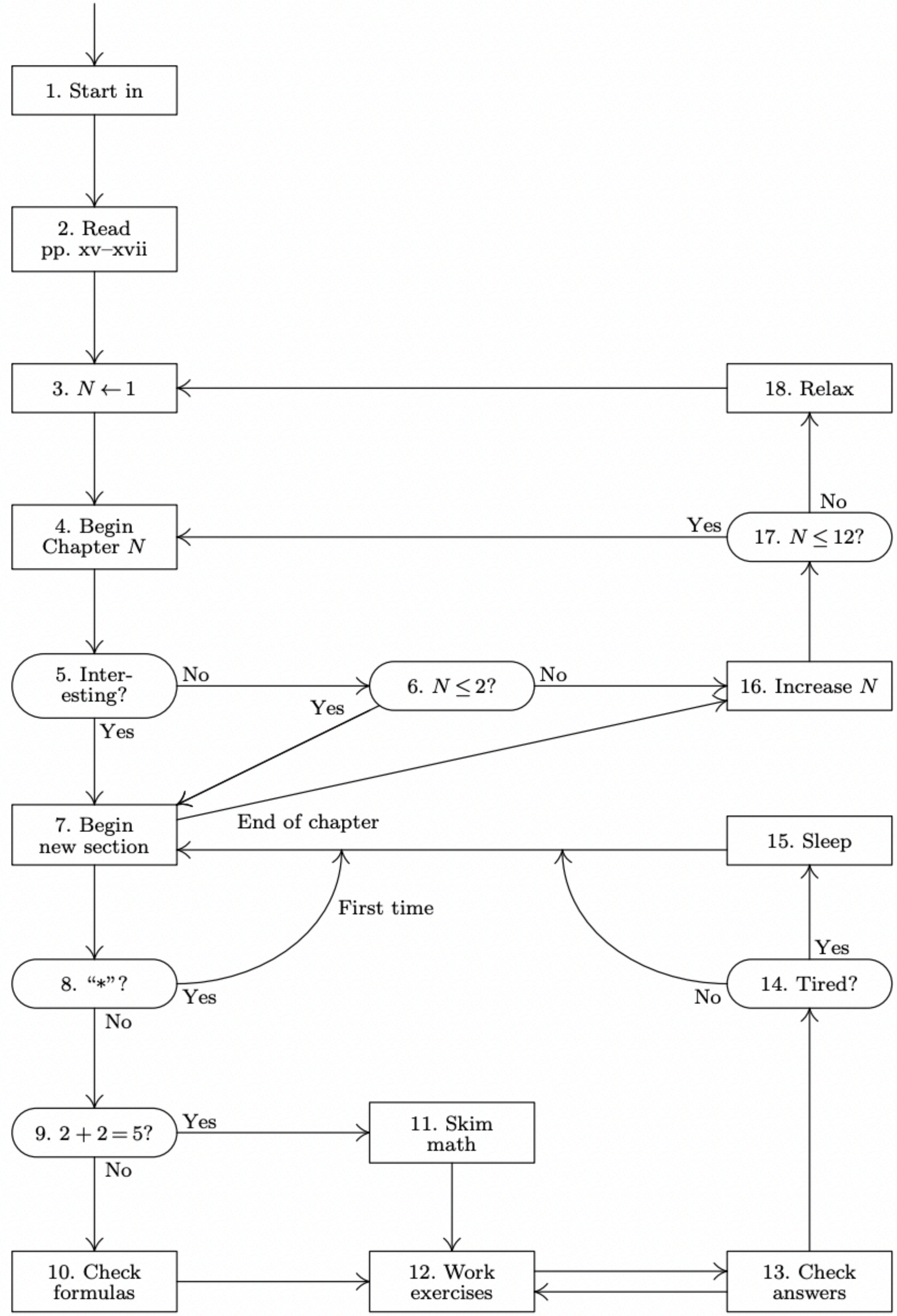
- The Art of Computer Programming,
    - Vol. 1: Fundamental Algorithms
    - Vol.2. Seminumerical Algorithms
    - Vol. 3: Sorting and Searching
    - (WIP) Vol. 4: Combinatorial Algorithms, Part 1
    - (WIP) Vol. 5: Syntactic Algorithms
    - (WIP) Vol. 6: The Theory of Context-free Languages
    - (WIP) Vol. 7: Compiler Techniques



Flow chart for reading this set of books.

The Art of Computer Programming

- Book about algorithms that has algorithm describing how to read it.
- “I retired early because I realized that I would need at least 20 years of full-time work to complete The Art of Computer Programming (TAOCP), which I have always viewed as the most important project of my life.”
- "My full-time writing schedule means that I have to be pretty much a hermit. The only way to gain enough efficiency to complete The Art of Computer Programming is to operate in batch mode, concentrating intensively and uninterruptedly on one subject at a time, rather than swapping a number of topics in and out of my head”
- I miss teaching and daily interaction with students, but I get stimulating feedback by giving regular public lectures and speaking to informal Stanford seminars.



Flow chart for reading this set of books.

Surreal Numbers

How two ex-students turned on to pure mathematics and found total happiness

...

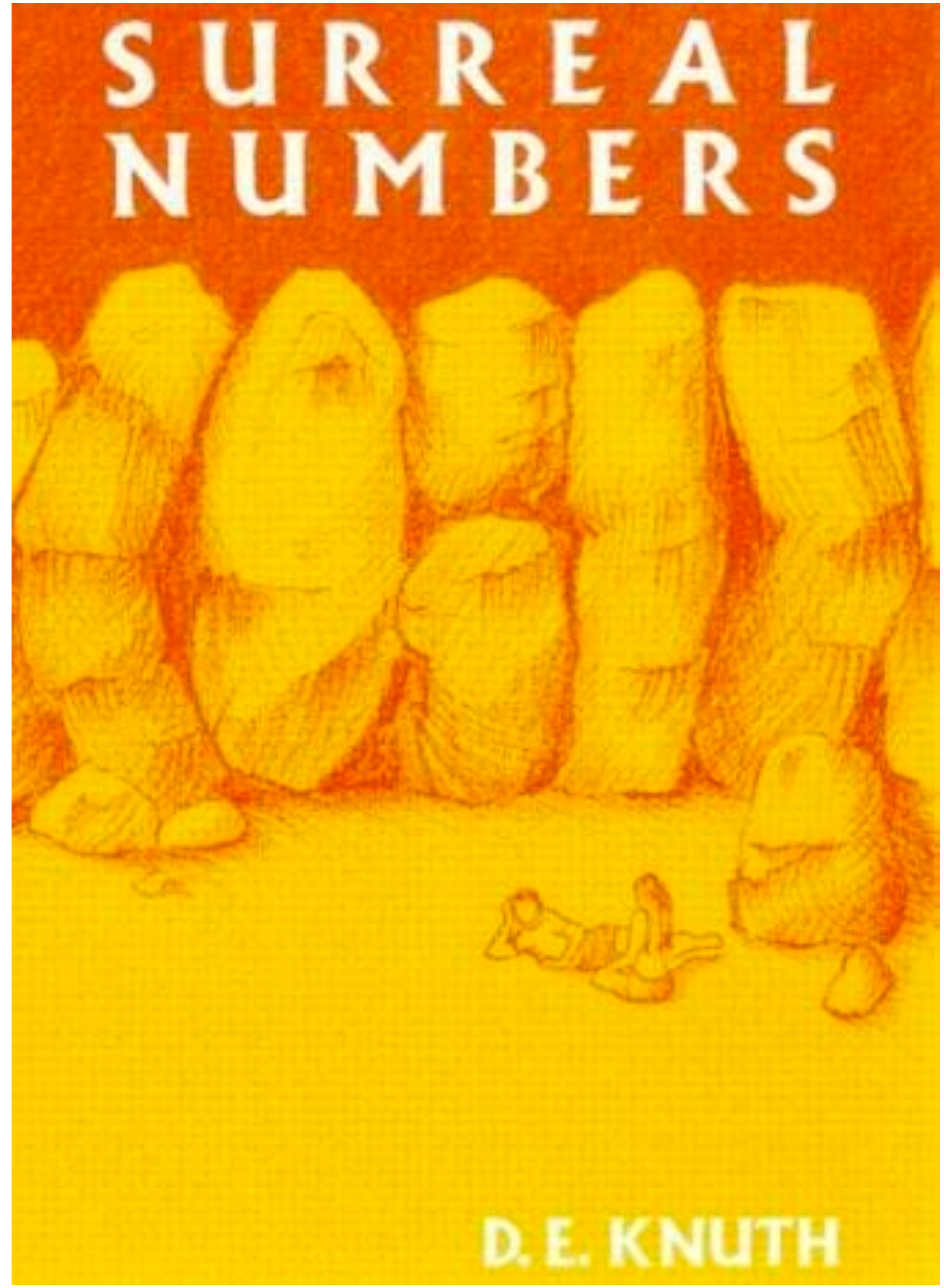
A. The Stone's version is a little different, but $x \leq y$ must mean the same thing as $y \geq x$.

B. Yeah, you're right. Hey, wait a sec, look here at these carvings off to the side:

● = ⟨ : ⟩
| = ⟨ ● : ⟩
— = ⟨ : ● ⟩

These are the symbols I couldn't decipher yesterday, and your notation makes it all crystal clear! Those double dots separate the left set from the right set. You must be on the right track.

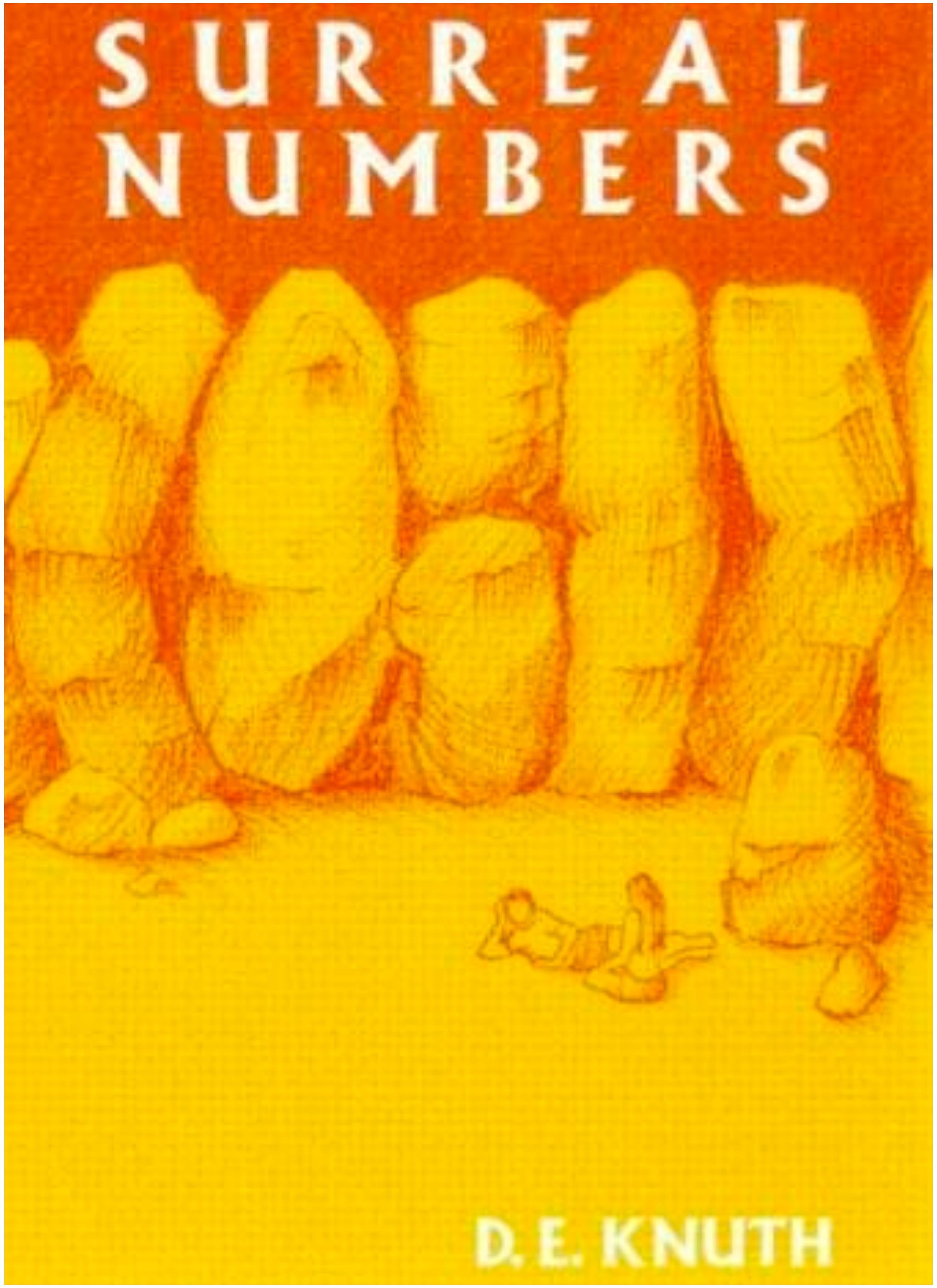
A. Wow, equal signs and everything! That stone-age carver must have used — to stand for -1; I almost like his notation better than mine.



Surreal Numbers

How two ex-students turned on to pure mathematics and found total happiness

- it is the only time a major mathematical discovery has been published first in a work of fiction. ... The book's primary aim, Knuth explains in a postscript, is not so much to teach Conway's theory as "to teach how one might go about developing such a theory." He continues: "Therefore, as the two characters in this book gradually explore and build up Conway's number system, I have recorded their false starts and frustrations as well as their good ideas. I wanted to give a reasonably faithful portrayal of the important principles, techniques, joys, passions, and philosophy of mathematics, so I wrote the story as I was actually doing the research myself."
- "There was a need to show how mathematical research is done, and how you take some simple laws and create gigantic universes out of simple laws. The process of discovering is usually not taught in schools."



TeX

- Is a typesetting system first released in 1978.
- Written in WEB/Pascal
- LaTeX is like standard library to TeX (created by Leslie Lamport).
- TeX is about the formatting.
- LaTeX is about the content.

```
Temperature reading were simulated using following
formula:
\[\textrm{temp}(d, h, m, rand) =
(10+\sin(\frac{\pi}{12} (h+\frac{m}{60}-10))) +
\frac{1}{2}\sin(d*\frac{\pi}{3.5}))
(1+\frac{rand-0.5}{70})\]
Humidity reading were simulated using following
formula: \[\textrm{humd}(hour, minute, rand) =
(60+\cos(\frac{\pi}{12}
(hour+\frac{minute}{60}-10)+\pi))
(1+\frac{rand-0.5}{70})\]
```

temperature (odd identifier) or humidity (even identifier). Temperature reading were simulated using following formula:

$$\textrm{temp}(d, h, m, rand) = (10+\sin(\frac{\pi}{12} (h+\frac{m}{60}-10))) + \frac{1}{2}\sin(d*\frac{\pi}{3.5}))(1+\frac{rand-0.5}{70})$$

Humidity reading were simulated using following formula:

$$\textrm{humd}(hour, minute, rand) = (60+\cos(\frac{\pi}{12}(hour+\frac{minute}{60}-10)+\pi))(1+\frac{rand-0.5}{70})$$

Knuth's up-arrow notation

- Is an alternative notation of representing hyperoperations.
- Hyperoperations is a sequence of arithmetic operations that generalise the notion of addition, multiplication, exponentiation.
- If:
  - Multiplication is iteration of additions.
  - Exponentiation is iteration of multiplications.
- Then
  - What is the iteration of exponentiation? 🤔
  - Iteration of what is addition?🤔

$$a \uparrow\uparrow\uparrow b = \left.\underbrace{a^{a^{\cdot^{\cdot^{a}}}}}_{\underbrace{a^{a^{\cdot^{\cdot^{a}}}}}_{\underbrace{\vdots}_{a}}}\right\} b$$

Knuth's up-arrow notation

- What is the iteration of exponentiation?🤔
  - Tetration.
  - What is the iteration of tetration? Pentation.
  - What ... ? Hexagon, etc.

$$a \uparrow\uparrow\uparrow b = \left.\underbrace{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}}_{\underbrace{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}}_{\underbrace{\vdots}_{a}}}\right\} b$$

Knuth's up-arrow notation

- Iteration of what is addition? 🤔
  - Addition is an iteration of **zeration** or successor function.
  - Successor function is a unary function that increments a given number.
  - Addition is an iteration of incrementations.

$$a \uparrow\uparrow\uparrow b = \left.\begin{matrix} \underbrace{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}} \\ \underbrace{a^{a^{\cdot^{\cdot^{\cdot^{a}}}}}} \\ \underbrace{\vdots} \\ a \end{matrix}\right\} b$$

Knuth's up-arrow notation

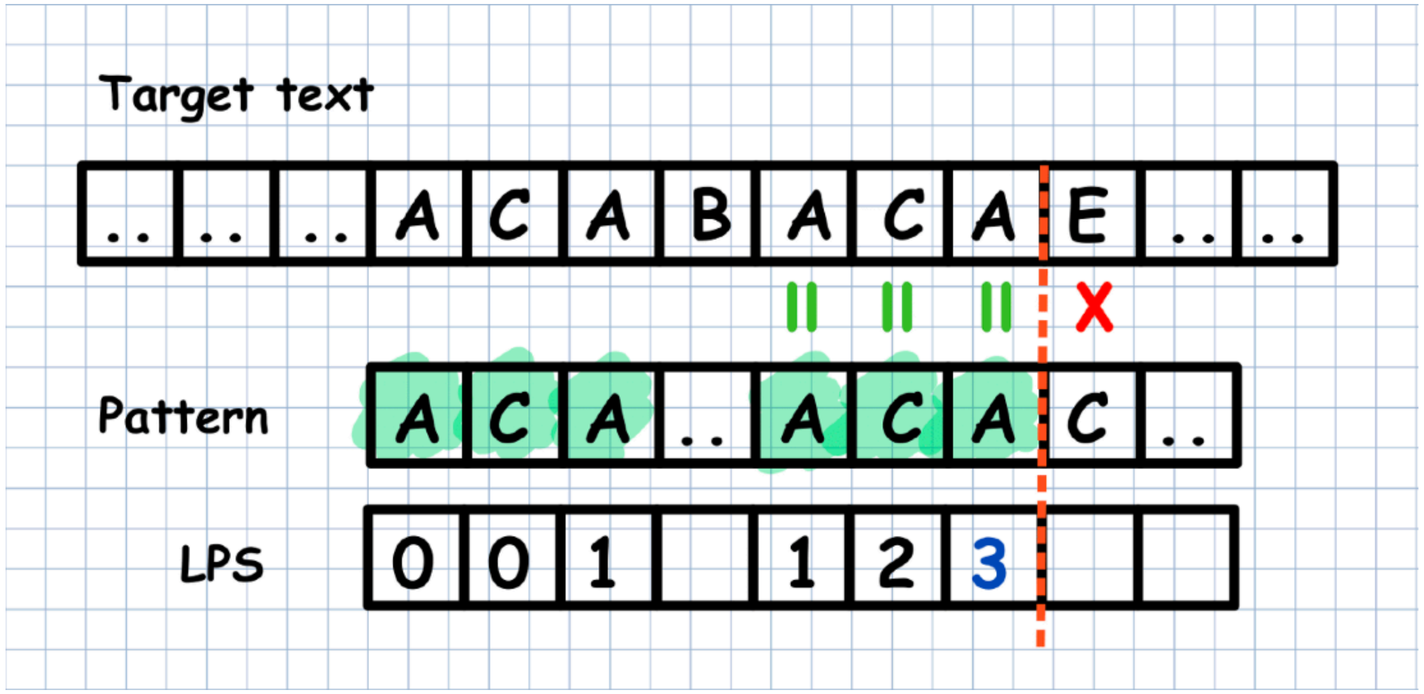
- How to represent hyperoperations?
  - square bracket notation: a[n]b
    - $a[0]b = b + 1$
    - $a[1]b = a + b = a + 1 + 1 + \ldots + 1$
    - $a[2]b = a \times b = a + a + \ldots + a$
    - $a[3]b = a^b = a \times a \ldots \times a = a\hat{}b$
      (where does the ^ comes from?🤔)
  - From Knuth's up-arrow notation
    - $a[3]b = a \uparrow b = a^b$
    - $a[4]b = a \uparrow \uparrow b = a^{a^{\cdot^{\cdot^{a}}}}$

$$a \uparrow\uparrow\uparrow b = \left.\underbrace{a^{a^{\cdot^{\cdot^{a}}}}}_{\underbrace{a^{a^{\cdot^{\cdot^{a}}}}}_{\underbrace{\vdots}_{a}}}\right\} b$$

Knuth–Morris–Pratt (KMP)

String search algorithm

- searches for occurrences of a "word" W within a main "text string" S by employing the observation that when a mismatch occurs, the word itself embodies sufficient information to determine where the next match could begin, thus bypassing re-examination of previously matched characters.
- complexity O(n+m), where n is the length of S and m is the length of W. Since $S \leq W$, KMP complexity is O(n).
- It preprocesses W, with the size complexity $O(m)$.



Knuth–Bendix

completion algorithm

- Algorithm for deciding whether two expressions are equivalent with respect to a set of rewriting identities.
- If they are equal $2+3 \stackrel{?}{=} 8+(-3)$
- If there is a substitution $2+x \stackrel{?}{=} 8+(-x)$

KNUTH–BENDIX COMPLETION

Given the four axioms

$$0+x = x \qquad \text{(A1)}$$
$$x+0 = x \qquad \text{(A2)}$$
$$-x+x = 0 \qquad \text{(A3)}$$
$$(x+y)+z = x+(y+z), \qquad \text{(A4)}$$

prove that $--a=a$ for any a.

Proof :-	1)	$--a$	=	$--a+0$	by A2
	2)		=	$--a+(-a+a)$	by A3
	3)		=	$(--a+-a)+a$	by A4
	4)		=	$0+a$	by A3
	5)		=	a	by A1

Figure 1. A typical proof of $--a=a$.

Proof :-	1)	$--a$	=	$--a+0$	by A2
	2)		=	$--a+(-x+x)$	by A3
	3)		=	$(--a+-x)+x$	by A4
	4)		=	$0+a$	by A3 with x instantiated to a
	5)		=	a	by A1

Figure 2. An alternative representation of the proof of $--a=a$.

My advice to young people

- “Don’t believe that just because something is trendy it’s good”
- “You won’t get prestige working on something popular, you get prestige by doing good science”
- "Follow your intuition, not the community”
- “Learn something about everything, and everything about something”

Donald Knuth - My advice to young people (93/97)